SADLER UNIT 4 MATHEMATICS METHODS

WORKED SOLUTIONS

Chapter 2 Calculus involving logarithmic functions

Exercise 2A

Question 1

$$\frac{1}{x}$$

Question 2

$$\frac{1}{2x}\times 2=\frac{1}{x}$$

Question 3

$$10x+\frac{1}{x}$$

Question 4

$$1+e^{x}+\frac{1}{x}$$

Question 5

$$\frac{3}{3x+2}$$

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Question 6

$$\frac{2}{2x+3}$$

Question 7

$$\frac{2}{2x-3}$$

Question 8

$$\frac{2x}{x^2+1}$$

Question 9

$$-\frac{\sin x}{\cos x}=-\tan x$$

Question 10

$$\frac{2x}{x^2}=\frac{2}{x}$$

Question 11

$$\frac{\frac{1}{3}x^{-\frac{2}{3}}}{x^{\frac{1}{3}}}=\frac{1}{3x}$$

Question 12

$$\frac{3\times\frac{1}{2}x^{-\frac{1}{2}}}{3\sqrt{x}}$$

$$=\frac{1}{2\sqrt{x}\times x^{\frac{1}{2}}}$$

$$=\frac{1}{2x}$$

Question 13

$$\frac{\frac{1}{5}}{\frac{x}{5}}=\frac{1}{x}$$

Question 14

$$\frac{2x+3}{x^2+3x}=\frac{2x+3}{x(x+3)}$$

Question 15

$$\frac{2x+1}{(x+4)(x-3)}$$

Question 16

$$x\times\frac{1}{x}+\ln x\times 1=\ln x+1$$

Question 17

$$3(\ln x)^2\times\frac{1}{x}=\frac{3(\ln x)^2}{x}$$

Question 18

$$\ln x^{-1}=-\ln x$$

$$\frac{d}{dx}(-\ln x)=-\frac{1}{x}$$

Question 19

$$\begin{aligned}\frac{d}{dx}(\ln x)^{-1}&=\frac{-1(\ln x)^{-2}\times 1}{x}\\&=-\frac{1}{x(\ln x)^2}\end{aligned}$$

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Question 20

$$e^x \times \frac{1}{x} + \ln x \times e^x = e^x\left(\frac{1}{x} + \ln x\right)$$

Question 21

$$\frac{x \times \frac{1}{x} - \ln x \times 1}{x^2}$$

$$= \frac{1 - \ln x}{x^2}$$

Question 22

$$3(1+\ln x)^2 \times \frac{1}{x} = \frac{3(1+\ln x)^2}{x}$$

Question 23

$$\frac{d}{dx}\left(\ln x + \ln(x+5) + \ln(x+3)\right)$$

$$= \frac{1}{x} + \frac{1}{x+5} + \frac{1}{x+3}$$

$$= \frac{(x+5)(x+3) + x(x+3) + x(x+5)}{x(x+5)(x+3)}$$

$$= \frac{3x^2 + 16x + 15}{x(x+5)(x+3)}$$

Question 24

$$\frac{d}{dx}\left(\ln(x+1) - \ln(x+3)\right)$$

$$= \frac{1}{x+1} - \frac{1}{x+3}$$

$$= \frac{(x+3) - (x+1)}{(x+1)(x+3)}$$

$$= \frac{2}{(x+1)(x+3)}$$

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Question 25

$$\frac{4(x^2+5)^3 \times 2x}{(x^2+5)^4} = \frac{8x}{x^2+5}$$

Question 26

$$\frac{d}{dx}\left(\ln x - \ln(x^2-1)\right)$$
$$= \frac{1}{x} - \frac{2x}{x^2-1}$$
$$= \frac{(x^2-1) - 2x \times x}{x(x^2-1)}$$
$$= \frac{-x^2-1}{x(x^2-1)}$$
$$= -\frac{(x^2+1)}{x(x^2-1)}$$
$$= \frac{x^2+1}{x(1-x^2)}$$

Question 27

$$\frac{d}{dx}\left(\ln(x+2)^3 - \ln(x-2)\right)$$
$$= \frac{d}{dx}\left(3\ln(x+2) - \ln(x-2)\right)$$
$$= \frac{3}{x+2} - \frac{1}{x-2}$$
$$= \frac{3(x-2) - 1(x+2)}{x^2-4}$$
$$= \frac{2x-8}{x^2-4}$$
$$= \frac{2(x-4)}{x^2-4}$$

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Question 28

$$y = 7\ln x$$
$$\frac{dy}{dx} = 7 \times \frac{1}{x}$$
$$= \frac{7}{x}$$

When $x = 1$,

$$\frac{dy}{dx} = 7$$

Question 29

$$y = x\ln x$$
$$\frac{dy}{dx} = x \times \frac{1}{x} + \ln x \times 1$$
$$= 1 + \ln x$$

When $x = e^2$,

$$\frac{dy}{dx} = 1 + \ln e^2$$
$$= 3$$

Question 30

$$y = 3x^2 + \ln x$$
$$\frac{dy}{dx} = 6x + \frac{1}{x}$$

When $x = 1$,

$$\frac{dy}{dx} = 6 + 1$$
$$= 7$$

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Question 31

$$y = -\frac{2\ln x}{x}$$

$$\frac{dy}{dx} = -\left(\frac{x \times \frac{2}{x} - 2\ln x \times 1}{x^2}\right)$$

$$= \frac{-2 + 2\ln x}{x^2}$$

When $x = 1$,

$$\frac{dy}{dx} = \frac{-2 + 2\ln 1}{1}$$

$$= -2$$

Question 32

$$y = \ln x$$

$$\frac{dy}{dx} = \frac{1}{x} = \frac{1}{4}$$

$$x = 4$$

When $x = 4$

$$y = \ln 4$$

$(4,\ \ln 4)$

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Question 33

$y = \ln(x^2)$

$\frac{dy}{dx} = \frac{2x}{x^2} = \frac{2}{x}$

$\frac{2}{x} = 4$

$x = \frac{1}{2}$

When $x = \frac{1}{2}$

$y = \ln\left(\frac{1}{4}\right)$

$= \ln 4^{-1}$

$= -1 \ln 4$

$\left(\frac{1}{2}, -\ln 4\right)$

Question 34

$y = \ln(6x - 5)$

$\frac{dy}{dx} = \frac{6}{6x-5} = \frac{6}{25}$

$6x - 5 = 25$

$6x = 30$

$x = 5$

When $x = 5$

$y = \ln 25$

$\therefore (5,\ \ln 25)$

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Question 35

$y = \ln(x^2 + 3x)$

$\frac{dy}{dx} = \frac{2x+3}{x^2+3x} = \frac{1}{2}$

$4x + 6 = x^2 + 3x$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3,\ x = -2 \qquad (x > 0)$

$x = 3$

When $x = 3$,

$y = \ln 18$

$\therefore (3,\ \ln 18)$

Question 36

$y = \ln x$

$\frac{dy}{dx} = \frac{1}{x}$

When $x = 1$

$\frac{dy}{dx} = 1$

Equation of tangent

$y = x + c$

Using (1,0)

$0 = 1(1) + c$

$c = -1$

$\therefore y = x - 1$

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Question 37

$y = \ln x$

$\frac{dy}{dx} = \frac{1}{x}$

When $x = e$

$\therefore \frac{dy}{dx} = \frac{1}{e}$

Equation of tangent

$y = \frac{x}{e} + c$

Using $(e, 1)$

$1 = \frac{1}{e} \times e + c$

$c = 0$

$\therefore y = \frac{x}{e}$

$ey = x$

Question 38

$$\begin{aligned} y &= \log_4 x \\ &= \frac{\ln x}{\ln 4} \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{\ln 4} \times \frac{1}{x} \\ &= \frac{1}{x \ln 4} \end{aligned}$$

Question 39

$$\begin{aligned} y &= \log_6 x \\ &= \frac{\ln x}{\ln 6} \end{aligned}$$

$$\begin{aligned} y' &= \frac{1}{\ln 6} \times \frac{1}{x} \\ &= \frac{1}{x \ln 6} \end{aligned}$$

Question 40

$$y = 50\ln x$$
$$\frac{dy}{dx} = 50 \times \frac{1}{x}$$
$$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$$
$$\delta y \approx \frac{50}{x} \times \delta x$$
$$\approx \frac{50}{10} \times 0.1$$
$$\approx 0.5$$

By comparison
$\ln 10.1 - 50\ln 10 = 0.4975$

Question 41

$$x = t + \ln t$$
$$v = \frac{dx}{dt}$$
$$= 1 + \frac{1}{t}$$
When $x = 2$,
$$v = 1 + \frac{1}{2}$$
$$= 1.5 \text{ m/s}$$

$$a = \frac{dv}{dt}$$
$$= -1t^{-2}$$
$$= -\frac{1}{t^2}$$
When $x = 2$
$$a = -\frac{1}{4} \text{ m/s}^2$$

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Question 42

$y = x^2 - 50\ln 2x,\ x > 0$

$$\frac{dy}{dx} = 2x - \frac{50}{x} = 0$$

$$2x = \frac{50}{x}$$

$$2x^2 = 50$$

$$x^2 = 25$$

$$x = \pm 5 \quad (x > 0)$$

$$x = 5$$

When $x = 5$,

$$y = 25 - 50\ln 10$$

$$\frac{d^2y}{dx^2} = 2 + \frac{50}{x^2}$$

When $x = 5$

$$\frac{d^2y}{dx^2} = 2 + \frac{50}{25} > 0$$

$\therefore (5,\ 25 - 50\ln 10)$ is a minimum turning point.

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Exercise 2B

Question 1

$$5\int \frac{1}{x}\,dx = 5\ln x + c$$

Question 2

$$4\int \frac{1}{x}\,dx = 4\ln x + c$$

Question 3

$$\int \left(x + \frac{2}{x} \right) dx = \frac{x^2}{2} + 2\ln x + c$$

Question 4

$$\frac{1}{2}\int \frac{2}{2x}\,dx = \frac{1}{2}\ln 2x + c$$

Question 5

$$\int \frac{2x}{x^2+1}\,dx = \ln(x^2+1) + c$$

Question 6

$$\int \left(x^2 + \frac{5}{x} \right) dx = \frac{x^3}{3} + 5\ln x + c$$

Question 7

$$\int \left(4x + e^x + \frac{2}{x} \right) dx = 2x^2 + e^x + 2\ln x + c$$

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Question 8

$$2\int \frac{1}{x+1}\,dx = 2\times \ln(x+1)+c$$

Question 9

$$4\int \frac{2x}{x^2-3}\,dx = 4\ln(x^2-3)+c$$

Question 10

$$\int \frac{5}{5x-3}\,dx = \ln(5x-3)+c$$

Question 11

$$5\int \frac{2}{2x+1}\,dx = 5\ln(2x+1)+c$$

Question 12

$$3\int \frac{2x}{x^2+1}dx = 3\ln(x^2+1)+c$$

Question 13

$$\int \frac{2x+1}{x^2+x+3}\,dx = \ln(x^2+x+3)+c$$

Question 14

$$3\int \frac{2x+5}{x^2+5x}\,dx = 3\ln(x^2+5x)+c$$

Question 15

$$10\int \frac{2x}{x^2+4}\,dx = 10\ln(x^2+4)+c$$

Question 16

$$-\int \frac{(-\sin x)}{\cos x}\,dx = -\ln(\cos x) + c$$

Question 17

$$\int \frac{\cos x}{\sin x}\,dx = \ln(\sin x) + c$$

Question 18

$$-\frac{1}{2}\int \frac{(-2\sin 2x)}{\cos 2x}\,dx = -\frac{1}{2}\ln(\cos 2x) + c$$

Question 19

$$\begin{aligned}\int \tan x\,dx &= \int \frac{\sin x}{\cos x}\,dx \\ &= -\ln(\cos x) + c\end{aligned}$$

Question 20

$$-\frac{1}{5}\int \frac{(-5\sin 5x)}{\cos 5x}\,dx = -\frac{1}{5}\ln(\cos 5x) + c$$

Question 21

$$-3\int \frac{(-2\sin 2x)}{\cos 2x}\,dx = -3\ln(\cos 2x) + c$$

Question 22

$$\begin{aligned}&\int \frac{\sin x - \cos x}{\sin x + \cos x}\,dx \\ &= -\int \frac{\cos x - \sin x}{\sin x + \cos x} \\ &= -\ln(\sin x + \cos x) + c\end{aligned}$$

$$\begin{aligned}&\frac{d}{dx}(\sin x + \cos x) \\ &= \cos x - \sin x \\ &= -(\sin x - \cos x)\end{aligned}$$

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Question 23

$$\int \frac{2+\cos 2x}{4x+\sin 2x}\,dx \qquad\qquad \frac{d}{dx}(4x+\sin 2x)=4+2\cos 2x$$

$$=\frac{1}{2}\int \frac{4+2\cos 2x}{4x+\sin 2x}\,dx$$

$$=\frac{1}{2}\ln(4x+\sin 2x)\,+\,c$$

Question 24

$$\int \frac{e^x+1}{e^x+x}\,dx \qquad\qquad \frac{d}{dx}(e^x+x)=e^x+1$$

$$=\ln(e^x+x)+c$$

Question 25

$$\begin{aligned}\int_1^3 \frac{1}{x}\,dx &= [\ln x]_1^3\\ &= \ln 3-\ln 1\\ &= \ln 3\end{aligned}$$

Question 26

$$\begin{aligned}\int_2^3 \frac{3}{x}\,dx &= [3\ln x]_2^3\\ &= 3\ln 3-3\ln 2\\ &= 3\ln 1.5\end{aligned}$$

Question 27

$$\begin{aligned}\int_1^2 \left(e^x+\frac{1}{x}\right)dx &= \left[e^x+\ln x\right]_1^2\\ &= e^2+\ln 2-e^1-\ln 1\\ &= e^2-e+\ln 2\end{aligned}$$

Question 28

$$v = \frac{1}{t+2}$$

$$x = \int v \ \ dt$$

$$= \int \frac{1}{t+2} \ dt$$

$$= \ln(t+2) + c$$

When $x = 0,\ t = 0$

$$0 = \ln 2 + c$$

$$c = -\ln 2$$

$$x = \ln(t+2) - \ln 2$$

$$= \ln \frac{(t+2)}{2}$$

Question 29

$$\int_1^3 \frac{2x+1}{x} \ dx = \int_1^3 2 \ dx + \int_1^3 \frac{1}{x} \ dx$$

$$= \left[2x\right]_1^3 + \left[\ln x\right]_1^3$$

$$= 6 - 2 + \ln 3 - \ln 1$$

$$= (4 + \ln 3) \ \text{units}^2$$

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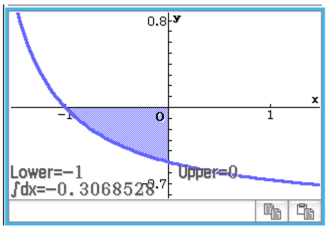
Question 30



$$y = \frac{1}{x+2} - 1$$

$$y\text{-int}: \frac{1}{2} - 1 = -\frac{1}{2} \qquad (0, -\frac{1}{2})$$

$$x\text{-int}: \frac{1}{x+2} = 1$$

$$x + 2 = 1$$

$$x = -1 \qquad (-1, 0)$$

$$\int_{-1}^{0} \left(\frac{1}{x+2} - 1 \right) dx = \left[\ln(x+2) - x \right]_{-1}^{0}$$

$$= \ln 2 - 0 - (\ln 1 + 1)$$

$$= \ln 2 - \ln 1 - 1$$

$$= \ln 2 - 1$$

This area is under x-axis. Therefore, the area is $(1 - \ln 2)$ square units.

Question 31

$$y = x\ln x - x$$

$$\frac{dy}{dx} = x \times \frac{1}{x} + \ln x \times 1 - 1$$

$$= \ln x$$

Point of Intersection

$$1 + \ln 2 = \ln x$$

$$1 = \ln x - \ln 2$$

$$1 = \ln \frac{x}{2}$$

$$1 = \log_e \frac{x}{2}$$

$$\therefore e^1 = \frac{x}{2}$$

$$x = 2e$$

x-intercept of $y = \ln x$

$$0 = \ln x$$

$$e^0 = x$$

$$x = 1$$

Required area

$$\int_1^{2e} \ln x \ dx$$

$$= \left[x\ln x - x\right]_1^{2e}$$

$$= \frac{(2e \times \ln 2e - 2e) - (1 \times \ln 1 - 1)}{2e(\ln 2 + \ln e) - 2e + 1}$$

$$= \frac{2e(\ln 2 + 1) - 2e + 1}{2e \times \ln 2 + 2e - 2e + 1}$$

$$= (2e\ln 2 + 1) \text{ units}^2$$

Question 32

$$\int_0^{\frac{\pi}{6}} \tan x \; dx$$

$$= \int_0^{\frac{\pi}{6}} \frac{\sin x}{\cos x} \; dx$$

$$= -\int_0^{\frac{\pi}{6}} \frac{(-\sin x)}{\cos x} \; dx$$

$$= \left[-\ln(\cos x)\right]_0^{\frac{\pi}{6}}$$

$$= -\ln \cos \frac{\pi}{6} - \left(-\ln(\cos 0)\right)$$

$$= -\ln \frac{\sqrt{3}}{2} - (-\ln 1)$$

$$= \ln\left(\frac{\sqrt{3}}{2}\right)^{-1}$$

$$= \ln\left(\frac{2}{\sqrt{3}}\right) \text{ units}^2$$

Question 33

$$\frac{a}{x+4}+\frac{b}{x+2}$$
$$=\frac{a(x+2)+b(x+4)}{(x+4)(x+2)}$$
$$=\frac{2(4x+13)}{(x+4)(x+2)}$$

$a(x+2)+b(x+4)=2(4x+13)$

$ax+2a+bx+4b=8x+26$

$a+b=8$ $\rightarrow$ Equation 1

$2a+4b=26$ $\rightarrow$ Equation 2

Solve simultaneously

$a=3$ and $b=5$

$$\frac{3}{x+4}+\frac{5}{x+2}=\frac{2(4x+13)}{(x+4)(x+2)}$$

$$\int\frac{2(4x+13)}{(x+4)(x+2)}\,dx$$
$$=\int\left(\frac{3}{x+4}+\frac{5}{x+2}\right)dx$$
$$=3\int\frac{1}{x+4}\,dx+5\int\frac{1}{x+2}\,dx$$
$$=3\ln(x+4)+5\ln(x+2)+c$$

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Question 34

a $\int_1^k \frac{2}{x}\,dx = 1$

$[2\ln x]_1^k = 2\ln k - 2\ln 1$

$1 = 2\ln k$

$\frac{1}{2} = \ln k$

$k = e^{\frac{1}{2}} \quad (k > 0)$

b $\int_1^b \frac{2}{x}\,dx = \frac{1}{2}$

$2\ln b = \frac{1}{2}$

$\ln b = \frac{1}{4}$

$b = e^{\frac{1}{4}}$

c $\int_1^{\frac{e^{\frac{1}{2}}+1}{2}} \frac{2}{x}\,dx$

$= [2\ln x]_1^{\frac{e^{\frac{1}{2}}+1}{2}}$

$= 2\ln\left(\frac{e^{\frac{1}{2}}+1}{2}\right) - 2\ln 1$

$= 2\ln\left(\frac{e^{\frac{1}{2}}+1}{2}\right)$

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Miscellaneous exercise two

Question 1

$$\frac{d}{dx}(\sin 2x) = 2\cos 2x$$

Question 2

$$\frac{d}{dx}(\cos 3x) = -3\sin 3x$$

Question 3

$$\frac{d}{dx}(e^{4x}) = 4e^{4x}$$

Question 4

$$\begin{aligned}\frac{d}{dx}(5e^{4x}) &= 5 \times 4e^{4x} \\ &= 20e^{4x}\end{aligned}$$

Question 5

$$\frac{d}{dx}\left(\frac{2x-3}{x+1}\right)$$

$$= \frac{(x+1)\times 2 - (2x-3)\times 1}{(x+1)^2}$$

$$= \frac{2x+2-2x+3}{(x+1)^2}$$

$$= \frac{5}{(x+1)^2}$$

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Question 6

$$\frac{d}{dx}(3x-1)^4$$
$$=4(3x-1)^3\times 3$$
$$=12(3x-1)^3$$

Question 7

$$\frac{d}{dx}(1+2\log_e x)$$
$$=2\times\frac{1}{x}$$
$$=\frac{2}{x}$$

Question 8

$$\frac{d}{dx}(x^2\ln x)$$
$$=x^2\times\frac{1}{x}+\ln x\times 2x$$
$$=x+\ln x\times 2x$$
$$=x(1+2\ln x)$$

Question 9

$$\frac{d}{dx}\left(\frac{1}{x}+3e^{2x}\right)$$
$$=-x^{-2}+3\times 2\times e^{x}$$
$$=6e^{2x}-\frac{1}{x^2}$$

Question 10

$$\frac{d}{dx}\left(\log_e(1+x+x^2)\right)$$
$$=\frac{1}{1+x+x^2}\times(1+2x)$$
$$=\frac{2x+1}{1+x+x^2}$$

Question 11

$$\begin{aligned}
2^x &= 11 \\
\log 2^x &= \log 11 \\
x \log 2 &= \log 11 \\
x &= \frac{\log 11}{\log 2}
\end{aligned}$$

Question 12

a

$$\begin{aligned}
&\log_a 25 \\
&= \log_a 5^2 \\
&= 2\log_a 5 \\
&= 2p
\end{aligned}$$

b

$$\begin{aligned}
&\log_a 500 \\
&= \log_a (5^3 \times 4) \\
&= \log_a 5^3 + \log_a 4 \\
&= 3\log_a 5 + \log_a 4 \\
&= 3p + q
\end{aligned}$$

c

$$\begin{aligned}
&\log_a 80 \\
&= \log_a (4^2 \times 5) \\
&= \log_a 4^2 + \log_a 5 \\
&= 2\log_a 4 + \log_a 5 \\
&= p + 2q
\end{aligned}$$

d

$$\begin{aligned}
&\log_a 10 \\
&= \log_a (5 \times \sqrt{4}) \\
&= \log_a 5 + \log_a 4^{\frac{1}{2}} \\
&= \log_a 5 + \frac{1}{2}\log_a 4 \\
&= p + \frac{1}{2}q
\end{aligned}$$

e

$$\begin{aligned}
&\log_a (20a^3) \\
&= \log_a 20 + \log_a a^3 \\
&= \log_a 4 + \log_a 5 + 3\log_a a \\
&= p + q + 3
\end{aligned}$$

f $\log_5 4$

$= \dfrac{\log 4}{\log 5}$

$= \dfrac{q}{p}$

Question 13

a $\log_x 64 = 3$

$x^3 = 64$

$x = 4$

b $\log_x 64 = 2$

$x^2 = 64$

$x = 8$

c $\log_x 64 = 6$

$x^6 = 64$

$x = 2$

d $\log_{10} 100 = x$

$10^x = 100$

$x = 2$

e $\log 17 - \log 2 = \log x$

$\log \dfrac{17}{2} = \log x$

$x = 8.5$

f $\log 17 + \log 2 = \log x$

$\log(17 \times 2) = \log x$

$x = 34$

g $\log 2^{\frac{1}{2}} = \log 2^x$

$2^{\frac{1}{2}} = 2^x$

$x = \dfrac{1}{2}$

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h $$\begin{aligned} 3\log 2 &= \log x \\ \log 2^3 &= \log x \\ x &= 2^3 \\ &= 8 \end{aligned}$$

Question 14

a $$\begin{aligned} \log_a x + \log_a y &= \log_a p \\ \log_a xy &= \log_a p \\ p &= xy \end{aligned}$$

b $$\begin{aligned} \log_x p &= y \\ p &= x^y \end{aligned}$$

c $$\begin{aligned} 3\log_a x - \log_a y &= \log_a p \\ \log_a x^3 - \log_a y &= \log_a p \\ \log_a \frac{x^3}{y} &= \log_a p \\ p &= \frac{x^3}{y} \end{aligned}$$

d $$\begin{aligned} 2 + 0.5\log_{10} y &= \log_{10} p \\ \log_{10} 100 + \log_{10} y^{0.5} &= \log_{10} p \\ \log_{10}\left(100y^{0.5}\right) &= \log_{10} p \\ p &= 100\sqrt{y} \end{aligned}$$

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Question 15

$$\frac{dy}{dx} = \frac{1}{x}$$

When $x = e^2$,

$$\frac{dy}{dx} = \frac{1}{e^2}$$

Equation of tangent

$$y = \frac{1}{e^2}x + c$$

Using $(e^2, 2)$

$$2 = \frac{1}{e^2} \times e^2 + c$$

$$2 = 1 + c$$

$$c = 1$$

$$y = \frac{1}{e^2}x + 1$$

$$e^2 y = x + e^2$$

Question 16

$$Q = Q_0(0.88)^t$$

$$0.05Q_0 = Q_0(0.88)^t$$

$$0.05 = 0.88^t$$

$$\log 0.05 = t \log 0.88$$

$$t = \frac{\log 0.05}{\log 0.88}$$

$$= 23.4 \text{ minutes}$$

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Question 17

a $f'(x) = 3x^2 \times \ln(3x+2)$

$$f''(x) = 3x^2 \times \frac{3}{3x+2} + \ln(3x+2) \times 6x$$

$$= \frac{9x^2}{3x+2} + 6x\ln(3x+2)$$

b $f''(1) = \frac{9}{5} + 6\ln 5$

$$= 6\ln 5 + 1.8$$

Question 18

a x-int, $y = 0$

$$(\log_e x)^2 - 1 = 0$$

$$(\log_e x)^2 = 1$$

$\log_e x = 1$ or $\log_e x = -1$

$x = e^1$ $\quad$ $x = e^{-1}$

x-int at $A\left(\frac{1}{e}, 0\right)$ and $B(e, 0)$.

There are no other possibilities for the graph to cut the x-axis.

If $y = (\log_e x)^2 - 1$ has a y-intercept, $x = 0$.

However, $\log_e x$ is not defined for $x = 0$, therefore there is no y-intercept.

b Stationary points occur when $\frac{dy}{dx}=0$.

$$\frac{dy}{dx}=2\log_e x\times\frac{1}{x}$$
$$=\frac{2}{x}\times\log_e x$$

$$\frac{2}{x}\log_e x=0$$

$$\log_e x=0$$
$$x=e^0$$
$$=1$$
$$y=(\log_e 1)^2-1$$
$$=-1$$

C$(1,-1)$

The single solution to $\frac{dy}{dx}=0$ shows there is only one stationary point.

c Point of inflection, $\frac{d^2y}{dx^2}=0$.

$$\frac{d}{dx}\left(\frac{2}{x}\log_e x\right)$$
$$=\frac{2}{x}\times\frac{1}{x}+\log_e x\times(-2x^{-2})$$
$$=\frac{2}{x^2}-\frac{2\log_e x}{x^2}$$

$$=\frac{2-2\log_e x}{x^2}$$

$$\frac{2-2\log_e x}{x^2}=0$$
$$2-2\log_e x=0$$
$$2\log_e x=2$$
$$\log_e x=1$$
$$x=e^1$$

$\therefore$ When $x=e$,

$$y=(\log_e e)^2-1$$
$$=0$$

There is a single point of inflection at B$(e,\ 0)$.

Question 19

$$= \lim_{h \to 0} \frac{\cos h - 1}{h}$$
$$= \lim_{h \to 0} \frac{-(1 - \cos h)}{h}$$
$$= -\lim_{h \to 0} \frac{1 - \cos h}{h}$$
$$= 0$$

$$\lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$
$$= \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$
$$= \lim_{h \to 0} \frac{\sin x(\cos h - 1) + \cos x \sin h}{h}$$
$$= \lim_{h \to 0} \frac{\sin x(\cos h - 1)}{h} + \lim_{h \to 0} \frac{\cos x \sin h}{h}$$
$$= \sin x \lim_{h \to 0} \frac{(\cos h - 1)}{h} + \cos x \lim_{h \to 0} \frac{\sin h}{h}$$
$$= \sin x \times 0 + \cos x \times 1$$
$$= \cos x$$

$$\lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$$
$$= \lim_{h \to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$$
$$= \lim_{h \to 0} \frac{\cos x(\cos h - 1) - \sin x \sin h}{h}$$
$$= \lim_{h \to 0} \frac{\cos x(\cos h - 1)}{h} - \lim_{h \to 0} \frac{\sin x \sin h}{h}$$
$$= \cos x \lim_{h \to 0} \frac{(\cos h - 1)}{h} - \sin x \lim_{h \to 0} \frac{\sin h}{h}$$
$$= \cos x \times 0 - \sin x \times 1$$
$$= -\sin x$$

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